# Selection and Performance

## CRITERIA FOR POWER TRANSMISSION COUPLINGS—PART II

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*"A flexible coupling, although it is relatively small and cheap compared to the machines it connects, is a critical aspect of any shaft system, and a good deal of attention must be paid to its choice at the design stage." —from the Resolution of the First International Conference on Flexible Couplings*

### Introduction

Part I of this article appeared in the October 2008 issue. It provided an overview and general classifications of power transmission couplings, along with selection and performance criteria for rigid couplings and misalignment-compensating couplings. Part II continues the discussion with selection and performance criteria for torsionally flexible and combination-purpose couplings.

### Torsionally Flexible Couplings and Combination Purpose Couplings

Torsionally flexible couplings usually have high torsional compliance (as compared with the torsional compliance of shafts and other transmission components) in order to enhance their influence on transmission dynamics. Figure 9a shows an example of a purely torsionally flexible coupling with an elastomeric flexible element having low stiffness in the torsional direction (shear of rubber ring) and high stiffness in the misalignment-compensation directions (compression of rubber ring). Figure 9b shows a torsionally flexible coupling with a metal flexible element (Bibby-style coupling). The flexible element is a spring steel band wrapped around judiciously shaped teeth on each hub and deforming between the teeth. The deformations become more restrained with increasing transmitted torque; thus the coupling has a strongly nonlinear torsional stiffness characteristic of the hardening type. The lowest stiffness is at zero torque (Fig. 9c) increasing towards the rated torque (Fig. 9d), becoming very high at the allowed peak torque (Fig. 9e) and approaching a rigid condition at an overload/shock torque (Fig. 9f). Since some misalignment-compensating ability is desirable for many applications, use of purely torsionally flexible couplings, with combination-purpose couplings being used as torsionally flexible couplings with more or less compensating ability.

For torsionally flexible and combination-purpose couplings, torsional stiffness is usually an indicator of payload capacity. In such cases, the basic design criterion can be formulated as a ratio between the stiffness in the basic misalignment direction and the torsional stiffness. In the following analysis, only radial misalignment is considered. Since couplings are often used as the cheapest connectors between shafts, and since end users often do not have full understanding of what is important for their applications, it is of interest to analyze what design parameters are important for various applications.

### Torsional Flexibility

Torsional flexibility is introduced into transmission systems when there is a danger of developing resonance conditions and/or transient dynamic overloads. Their influence on transmission dynamics can be due to one or more of the following factors: torsional compliance, damping or nonlinearity of load-deflection characteristics.

***Reduction of torsional stiffness of the transmission and, consequently, shift of its natural frequencies.*** If a resonance condition occurs before installation (or change) of the coupling, then shifting of natural frequency due to use of a high torsional-compliance coupling can eliminate resonance; thus dynamic loads and torsional vibrations will be substantially reduced. However, in many transmissions (e.g., vehicle transmissions), frequencies of the disturbances acting on the system and natural frequencies (especially in variable speed transmissions) may vary widely. In such instances, a simple shift of the natural frequencies of the drive may lead to a resonance occurring at other working conditions, but the probability of its occurrence is not lessened. A reduction in the natural frequency of a drive, for example, is advisable for the drive of a milling machine only at the highest spindle speeds and may be harmful if introduced in the low-speed stages.

A shift of natural frequencies of the drive may be beneficial in transmissions with narrow variations in working conditions. If, however, a drive is operated in the pre-resonance region, an increase in torsional compliance would lead to increased amplitudes of torsional vibrations, and thus to a nonuniform rotation. In some cases excessive torsional compliance may lead to a dynamic instability of the transmission and create intensive self-excited torsional vibrations.

An important feature of multispeed or variable-speed transmissions is the changing of effective torsional compliances of components with changing output speeds due to changing reduction coefficients, although the physical condition of the components does not change (Ref. 1). As a result, the role of the coupling as a compliant member can dramatically change depending on configuration of the drive. While compliance of a coupling of any reasonable size installed in the high-speed part of the system (close to the driving motor) would not have any noticeable effect at low output rpm, compliance of a coupling installed in the low-speed part of the system (close to the working organ, such as a wheel of the vehicle or a cutter of a mining combine) would be very effective, but the coupling size and cost might become excessive due to high

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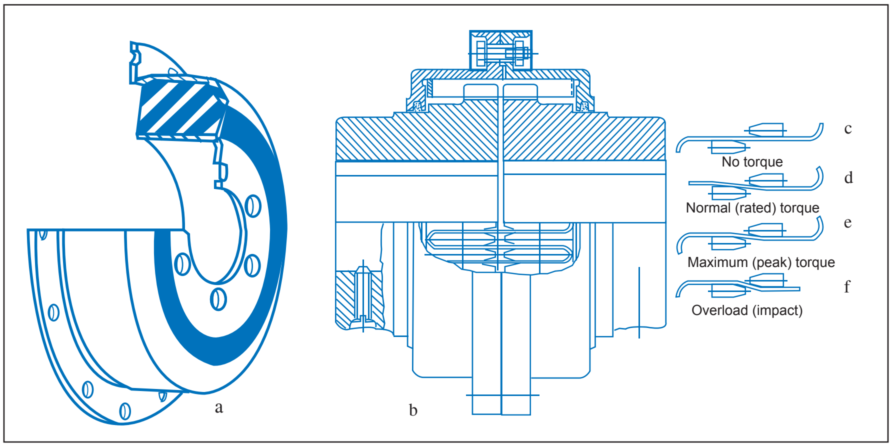


**Fig. 9—Torsionally-flexible, radially-rigid couplings: a) Dynaflex LCD, Lord Corp.; b) all-metal Bibby coupling.**

| Nomenclature | |
|---|---|
| $D$ | External diameter |
| $d$ | Internal diameter |
| $L$ | Length |
| $F_{com}$ | Radial force, or bending moment |
| $F_t$ | Tangential force |
| $R_{ef}$ | Effective radius |
| $T$ | Transmitted torque |
| $\mu$ | Friction coefficient |
| $k$ | Stiffness factor |
| $k_{com}$ | Combined stiffness of elastic connectors |
| $E$ | Radial misalignment |
| $D_p$ | Pitch Diameter |
| $\theta$ | Angular misalignment |
| $L_{eq}$ | Sound pressure level |
| $\eta$ | Efficiency |
| $k_{sh}$ | Shear stiffness |
| $\psi$ | Relative energy displacement |
| $V$ | Potential energy |
| $P_t$ | Tangential force |
| $W$ | Energy per coupling revolution |
| $\beta$ | Loss factor of rubber |

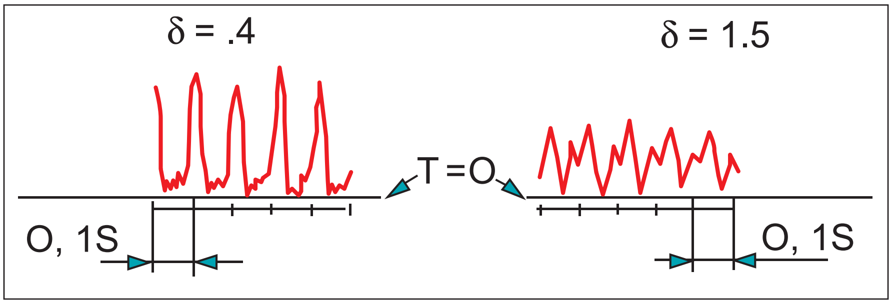


**Fig. 10—Dynamic load in a milling machine drive with: a) manufacturer-supplied motor coupling ($\delta$ = 0.4); b) high damping motor coupling ($\delta$ = 1.5).**

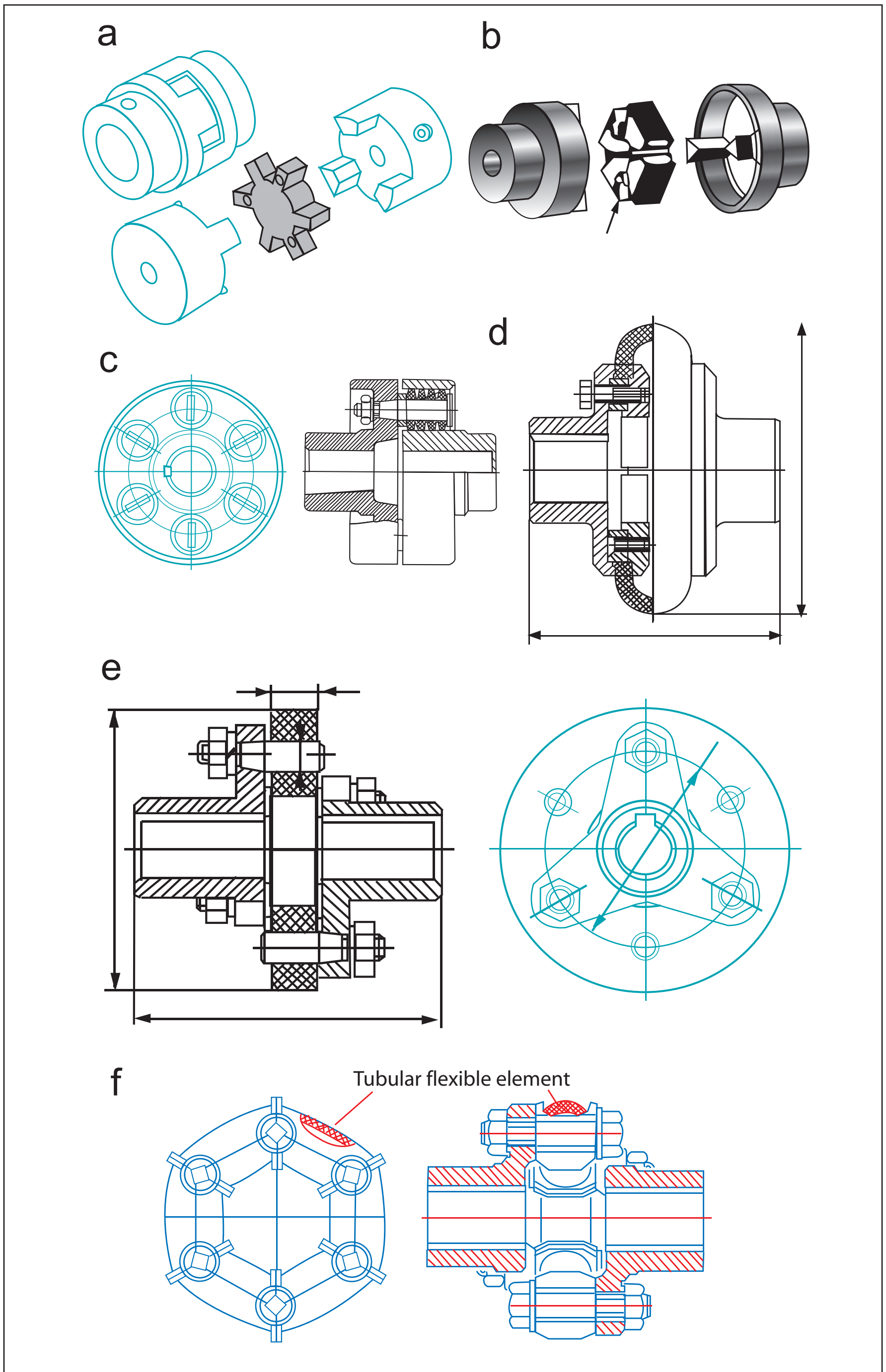


**Fig. 11—Some combination purpose couplings: a) spider (jaw) coupling; b) modified spider coupling (lip providing bulging space for the rubber element); c) finger-sleeve coupling; d) toroid shell coupling; e) rubber disc coupling; f) Centaflex coupling.**

torques transmitted to the spindle at low rpm.

***Increasing effective damping capacity of a transmission by using a high damping coupling or special dampers***. When the damping of a system is increased without changing its torsional stiffness, the amplitude of torsional vibrations is reduced at the resonance and in the near-resonance zones. Increased damping is especially advisable when there is a wide frequency-spectrum of disturbances acting on a drive; e.g., for the drives of universal milling machines. The effect of increased damping in a torsionally flexible coupling of a milling machine transmission is illustrated in Figure 10 (natural frequencies $f_{n1}$ = 10 Hz, $f_{n2}$ = 20 Hz). Figure 10a shows the resonance for an OEM coupling (flexible element made from neoprene rubber, log decrement $\delta \approx 0.4$). After this element was made from a butyl rubber (same compliance, but $\delta \approx 1.5$), the peak torque amplitude was reduced by ~ 1.8 times, the clearance opening (source of intensive noise) was eliminated, and oscillations with $f_{n2}$ excited by the second harmonic of the excitation force, became visible (Fig. 11b). A common misconception about using high-damping elastomers for coupling elements is their alleged high heat generation at resonance. Due to vibratory torque amplitude reduction with high-damping couplings, the heat generation at the resonance is *decreasing* when the high-damping coupling is used (Ref. 1). The influence of a flexible element on the total energy dissipation in a transmission increases with increasing of its damping capacity, of the torque amplitude in the element and of its compliance. For maximum efficiency, the flexible element of a coupling must therefore have as high an internal energy dissipation as possible; it must also possess maximum permissible compliance, and must be located in the part of the system where the intensity of vibrations is the greatest.

***Introducing nonlinearity in the transmission system***. A nonlinear dynamic system may automatically detune away from resonance at a fixed-frequency excitation. For example, when damping is low, a relative change of the stiffness by a factor of 1.3 reduces the resonance amplitude by ~1.7 times, but a relative change of stiffness by a factor of 2 reduces the resonance amplitude by ~1.85 times. Thus, nonlinear torsionally flexible couplings can be very effective in transmissions where high-intensity torsional vibrations may develop and where the coupling compliance constitutes a major portion of the overall compliance.

Vehicles usually have variable speed transmissions. The same is often true for production machines. In order to keep the coupling size small, it is usually installed close to the driving motor/engine, where it rotates with a relatively high speed and transmits a relatively small torque. At the lower speeds of an output member, the installed power is not fully utilized and the absolute values of torque (and of amplitudes of torsional vibrations) transmitted by the high-speed shaft are small. In vehicles, the installed power is not fully utilized most of the time. Thus, an important advantage of couplings with nonlinear load-deflection characteristics is feasibility of making a resonably small coupling with low torsional stiffness and high rated torque. An overwhelming majority of power transmission systems are loaded with less than 0.5 Tr for 80–90% of the total "up" time. A nonlinear coupling with a hardening load-deflection characteristic such as one in Figure 14 provides low torsional stiffness for most of the time, but since its stiffness at the rated torque is much higher, its size can be relatively small.

**Compensation Ability of Combination-Purpose Couplings.**

A huge variety of combination-purpose couplings is commercially available. Unfortunately, selection of a coupling type for a specific application is often based not on an assessment of performance characteristics of various couplings, but on the coupling cost or other non-technical considerations. As a result, bearings of the shafts connected by the coupling may need to be more frequently replaced than when an optimized coupling is used; the device might be noisier than it would be with a coupling type optimal for the given application, etc.

Figure 11 shows some popular designs of combination-purpose couplings.

Combination-purpose couplings do not have a compensating member. As a result, compensation of misalignment is accomplished, at least partially, by the same mode(s) of deformation of the flexible element as used for transmitting the payload. To better understand the behavior of combination-purpose couplings, an analysis of the compensating performance of a typical coupling with a spider-like flexible element is helpful. The coupling in Figure 12 shows a schematic of the jaw coupling in Figure 11a. It consists of hubs 1 and 2 connected with a rubber spider 3 having an even number $Z = 2n$ of legs, with "$n$" legs ("$n$" might be odd) loaded when hubs are rotating in the forward direction and the other $n$ legs loaded during the reverse rotation. Deformation of each leg is independent. The radial (compensation) stiffness of the coupling with $Z = 4$ is

$$k_{com} = \frac{F}{e} = 2k_t\sqrt{\cos^2\alpha + \frac{k_r^2}{k_t^2}\sin^2\alpha} \qquad (15)$$

where $F$ is radial force caused by the radial misalignment $e$ and acting on the connected shafts, $k_t$ is stiffness of one leg in compression (tangential direction), $k_r$ is stiffness in shear (radial direction), and $\alpha$ is angle of rotation of the coupling. Equation 15 shows that the total radial force $F$ fluctuates both in magnitude and in direction during one revolution.

For a coupling with $Z \geq 6$,

$$k_{com} = \frac{F}{e} = \frac{n}{2}(k_t + k_r) \qquad (16)$$

or $F$ is constant and is directed along the misalignment vector.

The ratio $k_r/k_t$ varies with changing rubber durometer $H$, and for typical spider proportions, $k_r/k_t$ = 0.26–0.3 for medium $H$ = 40–50, and $k_r/k_t$ = 0.4 for hard rubber spiders, $H$ = 70–75.

A spring/tubular spider coupling modification is shown in Figure 8 (Ref. 4). In this design, each leg of the spider can be represented by a coil spring loaded radially by the transmitted torque. For the tightly coiled extension spring $k_t$;

$$k_t \approx \frac{\pi^2 End^4}{3.74\,D^3}; \quad k_r = \frac{Gd^4}{8D^3 n}, \qquad (17)$$

thus $k_r/k_t = 1/53n^2 = 0.02/n^2 \approx 0$.

The torsional stiffness of both spider coupling designs is

$$k_{tor} = nk_t R_{eff}^2, \qquad (18)$$

where the effective radius $R_{eff}$ = ~$0.75R_{ex}$ = $0.75(D_{ex}/2)$. The ratios between torsional and compensation stiffness values are as follows:

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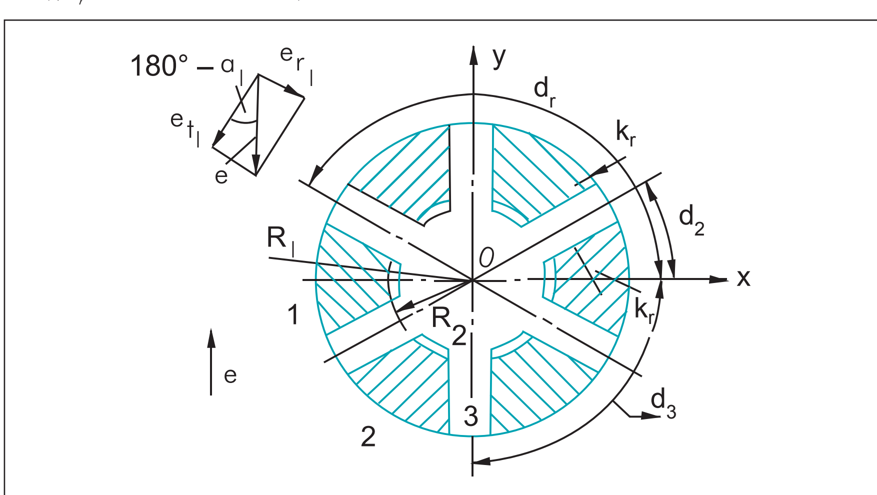


**Fig. 12—Schematic of a spider coupling: hubs (1, 2) and rubber spider (3).**

for $Z = 4$, $\left(\frac{k_{com}}{k_{tor}}\right)_{max} = \frac{1}{R_{eff}^2} \approx \frac{1.8}{R_{ex}^2}$; (18a)

for $Z \geq 6$, $\frac{k_{com}}{k_{tor}} = \frac{1.15}{R_{ex}^2}, H =$ $40-50$; $\frac{k_{com}}{k_{tor}} = \frac{1.25}{R_{ex}^2}, H = 65-75$;

for the spring spider coupling per Figure 8, $Z = 6$,

$$\frac{k_{com}}{k_{tor}} = \frac{0.9}{R_{ex}^2} \qquad (18b)$$

In general, the ratio of radial (compensating) stiffness and torsional stiffness of a combination-purpose flexible coupling can be represented as

$$\frac{k_{com}}{k_{tor}} = \frac{A}{R_{ex}^2}, \qquad (19)$$

where the "Coupling Design Index" $A$ allows one to select a coupling design better suited to a specific application. If the main purpose is to reduce misalignment-caused loading of the connected shafts and their bearings for a given value of torsional stiffness, then the least value of $A$ is the best, together with a large external radius. If the main purpose is to modify the dynamic characteristics of the transmission, then minimization of $k_{tor}$ is important.

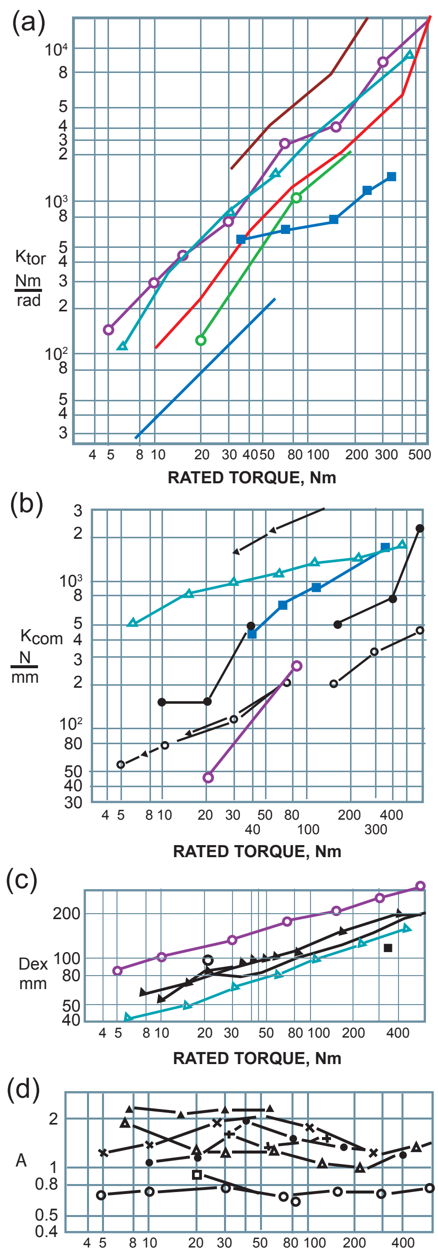


**Fig. 13—Basic characteristics of some combination-purpose couplings; a) torsional stiffness; b) radial stiffness; c) external diameter; d) coupling design index. ▲ - spider coupling in Fig. 10b; Δ - spider coupling with straight rectangular legs; + - finger sleeve coupling; o – toroid shell (tire) coupling; □ – rubber disc coupling; – Centaflex coupling; • – test results for coupling in Fig. 12 (same coupling at various transmitted torque).**

### Comparison of Combination Coupling Designs.

The bulk of designs of torsionally flexible or combination-purpose couplings employ elastomeric (rubber) flexible elements. Couplings with metal springs possess the advantages of being more durable and of having characteristics less dependent on frequency and amplitude of torsional vibrations. However, they may have a larger number of parts and higher cost, especially for smaller sizes. As a result, couplings with metal flexible elements, as of now, have found their main applications in large transmissions, usually for rated torques 1,000 N-m and up. Use of the modified spider coupling in Figure 8 may change this situation.

Couplings with elastomeric flexible elements can be classified in two subgroups:

(*a*) Couplings in which the flexible element contacts each hub along a continuous surface (shear couplings as in Figure 9a, toroidal shell couplings, couplings with a solid rubber disc/cone, etc.). Usually, torque transmission in these couplings is accommodated by shear deformation of rubber;

(*b*) Couplings in which the flexible element consists of several independent or interconnected sections (rubber disk and finger sleeve couplings as in Figure 11c, spider couplings as in Figures 11a and 11b, couplings with rubber blocks, etc.). Usually, torque transmission in these couplings is accommodated largely by compression or "squeeze" of rubber; thus they are usually smaller for a given rated torque.

Comparative evaluation of the commercially available couplings based on available manufacturer-supplied data on flexible couplings is presented in Figure 13. Plots in Figures 13a–d give data on torsional stiffness $k_{tor}$, radial stiffness $k_{rad}$, external diameter $D_{ex}$ , and design index $A$.

The "modified spider" coupling in Figure 11b is different from the conventional spider coupling shown schematically in Figure 11a by four features: its legs are tapered, instead of uniform width, and made thicker even in the smallest cross section, at the expense of reduced thickness of protrusions on the hubs; lips on the edges provide additional space for bulging of the rubber when the legs are compressed; and the spider is made of a very soft rubber. These features substantially reduce stiffness values while retaining the small size characteristic of the spider couplings.

Data for "toroid shell" couplings in Figure 13 are for the coupling as shown in Figure 10d.

The "spider coupling" for $T_r$ = 7 N-m has the number of legs $Z$ = 4 while larger sizes have $Z$ = 6 or 8. This explains differences in $A$ ($A$ = 1.96, close to theoretical 1.8, for $Z$ = 4; $A$ = 0.98–1.28, close to theoretical 1.15–1.25, for $Z$ = 6 or 8).

Values of A are quite consistent for a given type of coupling. Some variations can be explained by differences in design proportions and rubber blends between the sizes.

Plots in Figure 13 help to select a coupling type best suited for a particular application, but do not address issues of damping and nonlinearity. Damping can be easily modified by proper selection of the elastomer. High damping is beneficial for transmission dynamics, and may even reduce thermal exposure of the coupling.

A coupling with a hardening nonlinear characteristic may have high torsional compliance for the most frequently used sub-rated (fractional) loading in a relatively small coupling. Accordingly, the misalignment-compensating properties of a highly nonlinear coupling would be superior at fractional loads. The coupling in Figure 14 (Ref. 5) employs radially compressed rubber cylinders for torque transmission in one direction (117) and for the opposite direction (118), between hubs (111 and 113) attached to the connected shafts. This design combines the desirable nonlinearity with a significantly smaller size for a given $T_r$ (due to the use of multiple cylindrical elements with the same relative compression in each space between protruding blades, and due to high allowable compression of the rubber cylinders (Ref. 6), thus allowing use of smaller diameters and, consequently, many sets of cylinders around the circumference). Test results for such coupling for $T_r$ = 350 Nm are shown as ■ in Figure 13; in this case the data does not refer to different $T_r$, but to the same coupling at different transmitted torques.


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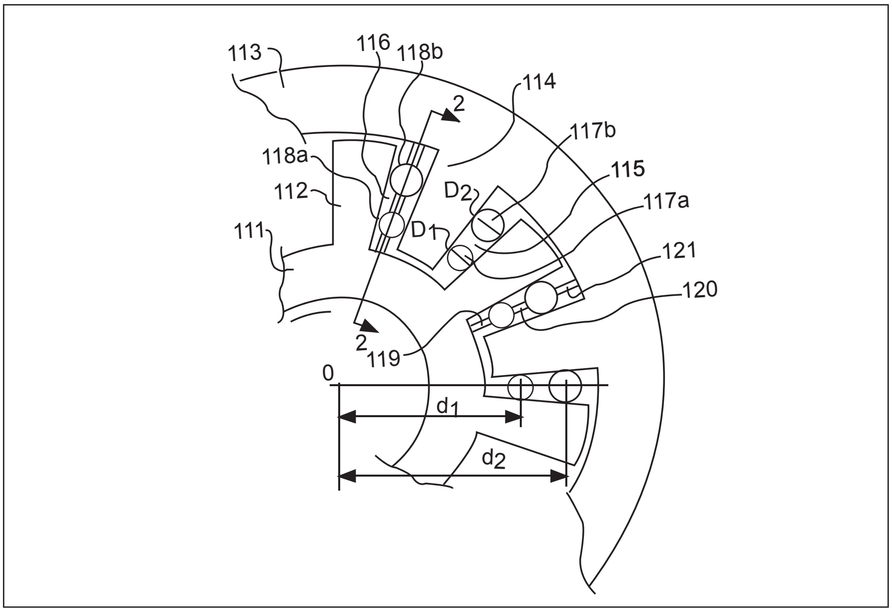


**Fig. 14—Torsionally flexible coupling with flexible element composed of radially compressed rubber cylinders.**


**Eugene I. Rivin** *was a Principal Staff Engineer at Ford Motor Co., from 1976–1981. Since 1981 he has been professor at Wayne State Univ. Major professional achievements of Dr. Rivin are in transmission dynamics, vibration/noise control, machine tools/tooling, robotics, advanced machine elements, creative problem solving. He published many monographs, book chapters and articles. Most recent books: "Mechanical Design of Robots," 1988; "Stiffness and Damping in Mechanical Design," 1999; "Passive Vibration Isolation," 2003; "Innovation on Demand," 2005 (with V. Fey). He authored/co-authored 60+ patents, with some inventions widely implemented worldwide. Out of this number, 18 patents relate to power transmission components (gears, flexible and rigid couplings, keys and other rigid interfaces/connections for machine tools and other mechanical systems). He is an elected Fellow of the International Academy of Production Engineering Research (CIRP), of ASME, and of SME.*