The Latest Design Technologies For Gear Devices with Great Transmission Ratios

Shuting Li

Four types of gear devices with great transmission ratios (simply called great ratio gears or GRGs) are discussed in this paper. They are strain wave gearing devices (SWGs), trochoidal gear reducers (TGRs), hypocyclic gear reducers (HGRs) and James Ferguson-type planetary drives (JFDs). The structures, advantages and basic performances of these four devices are compared. The latest design and strength analysis methods are also introduced. To conclude, the future tendencies of GRGs are predicted.

Introduction

In the latter period of the 20th century, factory automation was developed very rapidly in order to satisfy the requirements of mass production and reduce product cost. Industrial robots were successfully developed — mainly in Japan, Germany and Switzerland — for the express purpose of factory automation. Then GRGs — such as the SWGs and the TGRs — found applications in industry as robot joints. This was the beginning of GRGs being widely used in industry. Today, countless SWGs and TGRs are made every day, and they are used not only in robots, but also in many other high-performance machines such as semiconductor devices, aircraft and space-exploring machines, as well as critical power transmission devices.

Yet, after much R&D in making high-performance SWGs and TGRs a reality, design and strength calculation problems of the GRG remain.

This paper introduces the latest results of an extended period of comprehensive research on the design and strength calculation methods of the SWGs, TGRs, HGRs and JFDs. First, the structures, advantages and basic performance of the four GRGs are compared. Next, design and strength analysis methods developed for these four GRGs are introduced. And last, future tendencies are predicted for GRGs.

Structures, Advantages and Basic Performances of the GRGs

An SWG is also known as a harmonic drive device; it was invented by Musser (Ref. 1) in 1957. Figure 1 is a structural example of a cup-typed SWG. As shown there, the SWG mainly consists of a thin-rimmed flex-spline (FS) spur gear; an internal spur gear, also known as a circular spline (CS); an elliptical-shaped cam, or wave generator (WG); and a flexible ball bearing (FB) that can be deformed freely by the WG.

Transmission ratio of the SWG — when the CS is fixed and the FS is used as output — can be calculated by (Z_1/N). Here, Z_1 is the tooth number of the FS; N is lobes of the WG. Usually, the WG assumes the shape of an ellipsoid; therefore $N=2$ at this time. If Z_1 is substantial, great transmission ratio can be available. Now, the SWG with transmission ratios among 30 and 200 are made as mass production products by makers.

Beyond the great ratio, benefits such as light weight, small size, very slight backlash and extremely high transmission accuracy are other advantages of this device. Also, the SWG can be made

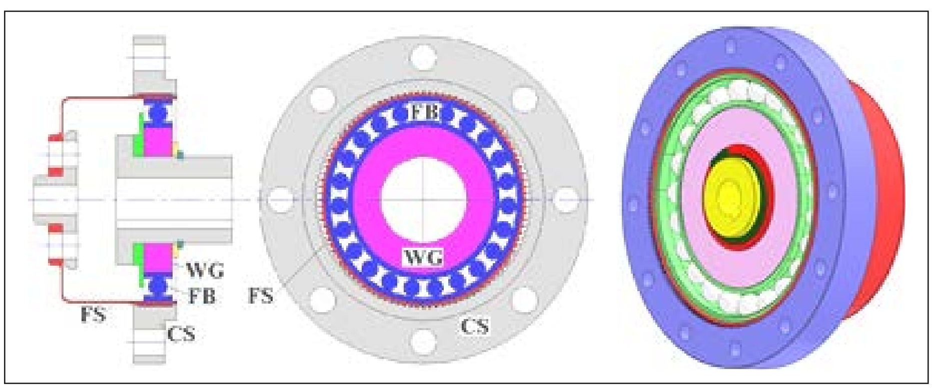

Figure 1 A structural example of the cup-type of SWG.

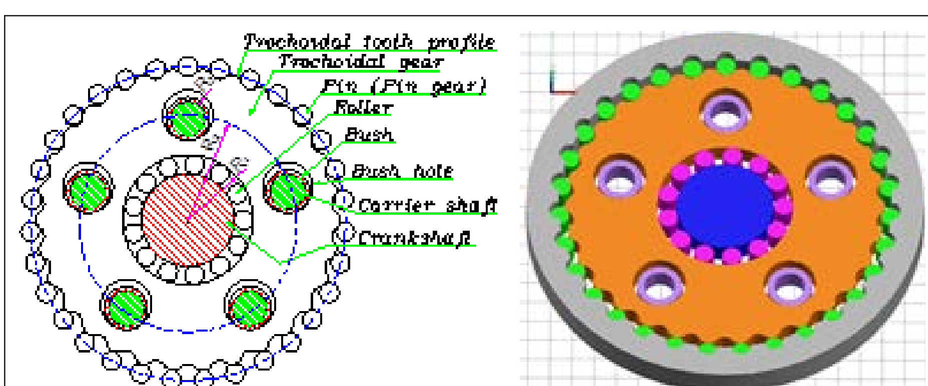

Figure 2 A structural example of the TGR.

very small — like a micro-machine. With these advantages, the SWG has found wide applications in industry, as well as in humanoid robots in recent years.

Trochoidal gear reducer (TGR). TGR is also known as a cycloidal gear reducer. It was invented by Lorenz Braren, a German engineer, in 1927 (Fig. 2). As shown there, an external spur gear with trochoidal profile — AKA *trochoidal* gear — is used. Many pins are used as teeth of an internal—or *pin*—gear. Like the SWG, an eccentric cam — or *crankshaft* — is used to drive the trochoidal gear to engage with the pin gear. Transmission ratio of the TGR can be calculated by $Z_2/(Z_2\text{-}Z_1)$ when the pin gear is used as output. Here, Z_1 and Z_2 are tooth numbers of the external and internal gears, respectively. Usually, $(Z_2 - Z_1) = 1$ and then the transmission ratio becomes Z_2. So, if Z_2 is made very large, extreme transmission ratio can be made available.

In addition to great transmission ratio, the TGR is also characterized by great torsional rigidity, high load-carrying and overloading capacities. This is because the number of contact teeth can be increased when larger torque occurs. The TGR also has high efficiency and strong tooth root strength because of the trochoidal profile. It is well known that only rolling contact (no sliding) exists between the contact teeth. But now the TGR can be made with very small backlash, lost motion and transmission errors.

Hypocyclic gear reducer (HGR). HGR is a planetary drive with small tooth number difference. Figure 3 is a structural example of the HGR. In Figure 3 it is seen that the kinematics of the HGR is exactly the same as for the TGR, with only difference being the tooth profile curves. When involute curves are used as tooth profiles of the external and internal gears, the TGR becomes the HGR. The HGR can be made simpler than the TRG. This is because it is simpler to make the involute teeth than to make the trochoidal teeth. Transmission ratio of the HGR can be calculated in the same way as the TGR. The HGR has lower load-carrying capacity than the TGR. This is because the number of contact teeth of the HGR is less than that of the TGR due to use of the involute profile. Of course, the torsional rigidity and overloading ability of the HGR are also lower than the TGR; perhaps this is the main reason why the HGR cannot be used widely in industry as is the TGR. It is presently a research subject of the author's as to how to increase the number of contact teeth of the HGR by way of a suitable involute tooth profile.

James Ferguson-type planetary drive (JFD). A James Ferguson mechanism (Ref. 2) is one in which one gear engages with the other two or three gears on the same tooth surface. This mechanism was also used in planetary drives for achieving great transmission ratio and compact size design (Ref. 3). The planetary drive that uses the James Ferguson mechanism is called the James Ferguson-type planetary drive (JFD) in this context (Fig. 4; Ref. 3). In Figure 4, Z_1 and Z_2 are tooth numbers of the sun gear and the planetary gear, respectively; Z_3 and Z_4 are tooth numbers of two internal gears. As shown in Figure 4, the planetary gear Z_2 engages with two internal gears (Z_3 and Z_4) on the same tooth surface. This is the main difference between a typical planetary drive and a JFD. In most cases a transmission ratio of a planetary drive cannot exceed 14; but the transmission ratio of the JFD can be greater than *100*. The transmission ratio of the JFD can be calculated by $(1 + Z_3/Z_1) / (1 - Z_3/Z_4)$. Here, Z_4 is greater than Z_3 and $(Z_4 - Z_3)$ is equal to the number of the planetary gears used in the JFD.

Tooth interference must be checked twice, since two internal gears are used in the JFD. In order to avoid tooth interference, Z_1, Z_2, Z_3 and Z_4 cannot be given freely as desired. So, too, transmission ratio of the JFD cannot be obtained freely as desired. This is the first disadvantage of the JFD. The second disadvantage of the JFD is lower load-carrying capacity. It shall be introduced later that tooth side-heavier contact occurred in the JFD due to a face width discrepancy between the planetary gears and internal gears. It, too, is now a continuing research project of the author on how to reduce the tooth side-heavier contact through considering a new structure, or by performing tooth lead modifications.

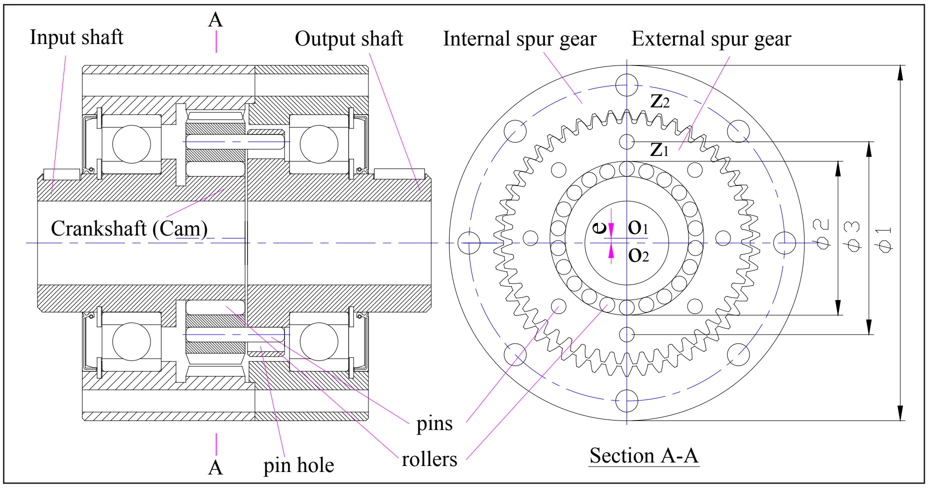

Figure 3 A structural example of the HGR.

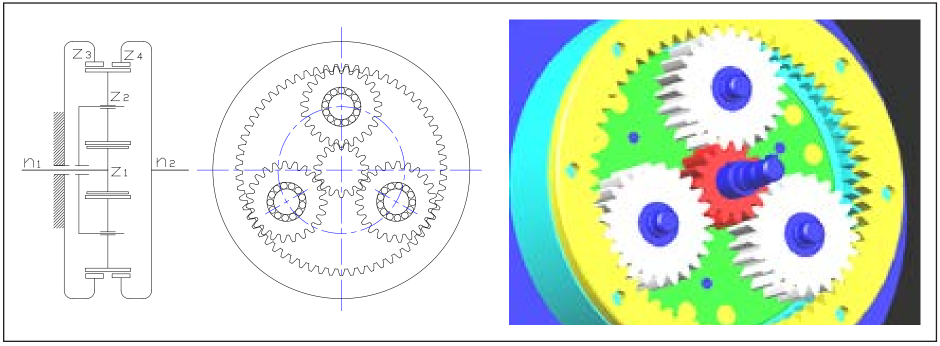

Figure 4 A structural example of the JFD.

The Latest Design and Strength Analysis Technologies for GRG

Design and strength analysis methods of the four GRGs differ from those of usual gears. For the latter, tooth contact and bending strength calculations are the main items in strength analyses, while for the GRG, tooth strength as well as bearing strength must be analyzed simultaneously. This is because contact fatigue failures of the bearings in the GRGs are also the *main* failures. In order to solve strength calculation problems of the GRG, special finite element methods have been developed to do contact analyses of these devices separately, based on long-term study by the author. Loads distributed on teeth, bearing balls and rollers are analyzed — initially through contact analysis with the developed FEM. Then tooth and bearing strength are evaluated using FEM and Hertz's formula once the loads on teeth, bearing balls and rollers become known. These main procedures are introduced in the following sections.

Strain wave gearing device (SWG). In the mass production products of the SWG, three kinds of curves are now used as tooth profiles; i.e. — straight line, involute, and arc profiles, respectively. Tooth profile design of the SWG is a difficult thing for designers. This is because tooth profile design of the SWG must take the cam shape of the WG into account since cam shape has direct effects on tooth engagement. It requires very sophisticated technology in order to be able to design a suitable tooth profile with high performances for the SWG. Special software has been developed to do tooth profile design and geometric dimension calculations of the SWG, based on more than 20 years' experience of the author with the SWG. Figure 5 is an example of applications of the developed software. This software was developed to run in the *AutoCAD* environment using *AutoCAD* VBA language. The straight line, involute, and arc curves are programmed in the software. When the gearing parameters are input in the software, tooth profile design and geometric dimension calculations can be conducted automatically with the help of this software. The 2D drawings (Fig. 5 (a)) can be drawn automatically on the drawing template of *AutoCAD* in a few seconds. 3D drawings (Fig. 1(b)) can also be executed via 3D commands of the *AutoCAD* or *SolidWorks* software based on the 2D drawings.

Strength calculations of the SWG are another difficult hurdle for designers. This is because there is not yet available a simple method to do strength calcula-

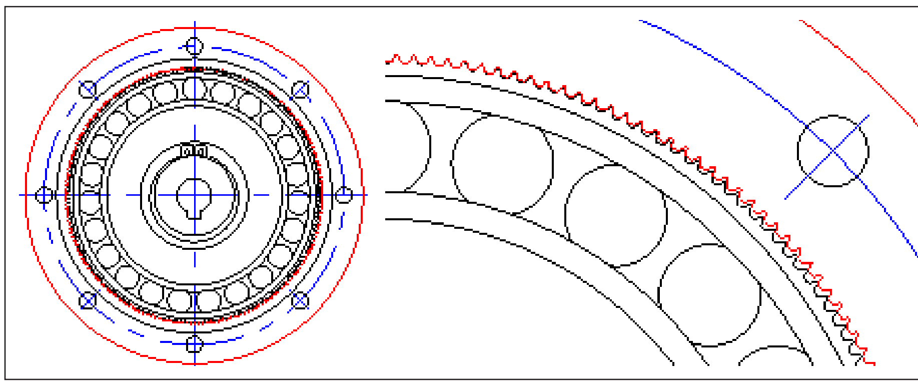

Figure 5 Geometric calculations and automatic drawing of the SWG.

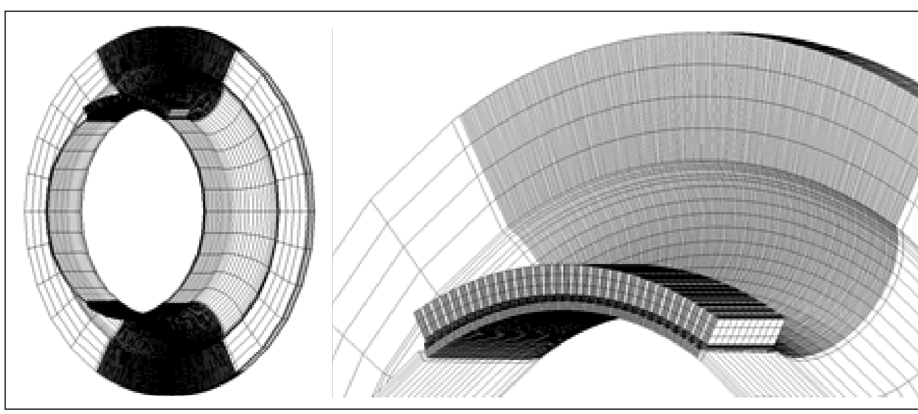

Figure 6 3D FEM model used for contact analysis of SWG.

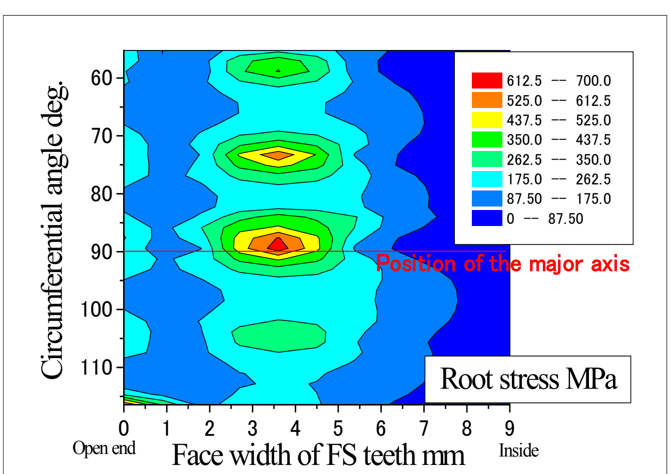

Figure 7 Tooth root stress distribution.

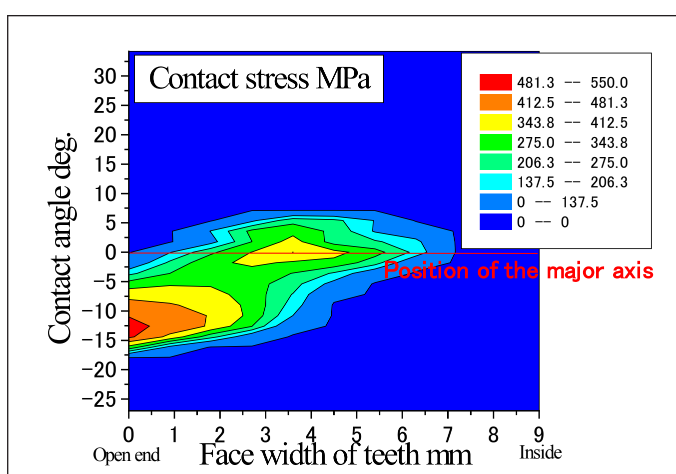

Figure 8 Tooth contact stress distribution.

tions of the SWG. To solve this problem the author in 1987 began seeking a suitable method for strength calculations of the SWG. A unique FEM for the SWG was eventually developed after a very long period of research. Today it is possible to calculate the following strength items of the SWG with the developed FEM software:

1. Contact strength of the contact teeth
2. Bending strength of FS tooth roots
3. Bending strength of FS structure (tube and diaphragm)
4. Contact strength of the FB.

(Except for these four items, buckling strength calculation of FS tube must also be included in the design procedures.)

Figure 6 is a 3D FEM model used for contact analysis of a silk-hat-type of the SWG (Ref. 4). Figure 7 is the root bending stresses of the FS teeth calculated by the developed FEM software. Figure 8 is the calculated maximum contact stresses on FS tooth surfaces. From Figure 7 it is found that the root bending stresses of the FS teeth are affected by the ball positions of the FB. Figure 8 indicates that the tooth contact pattern of the SWG is much more complex than that of usual gears.

Trochoidal gear reducer (TGR). Design software has been developed also for the TGR in an *AutoCAD* surrounding like the SWG (Fig. 9). With this software TGR can be designed very quickly and automatically. The 2D drawings of the designed TGR can be drawn automatically on the drawing template of *AutoCAD* (Fig. 2 (a)) and the 3D drawing can be made as shown (Fig. 2 (b)) using 3D commands of *AutoCAD* based on the 2D drawings.

Yet another difficulty is for designers to perform strength calculations of the TGR. This is because it is still an unsolved problem to do three-dimensional contact analyses of the loads on teeth, bushes and rollers of the TGR (Fig. 2(a)), although many efforts have been made to solve this problem. It was started in 1994 by the author to develop a method and software that can do strength analyses of the TGR using 2D FEM. Now it is indeed possible to calculate the contact loads and stresses on teeth, bushes and rollers of the TGR with the developed 2D FEM software.

Figure 10 is the 2D FEM model used for contact analysis of the TGR with the parameters indicated in Figure 9. Figures 11–13 are calculated contact loads and stresses on teeth, bushes and rollers when a torque load is applied. Numbers of the teeth, bushes and rollers used in Figures 11–13 are also indicated in Figure 10. With the calculated contact stresses, the contact strength of the teeth, bushes and rollers can be

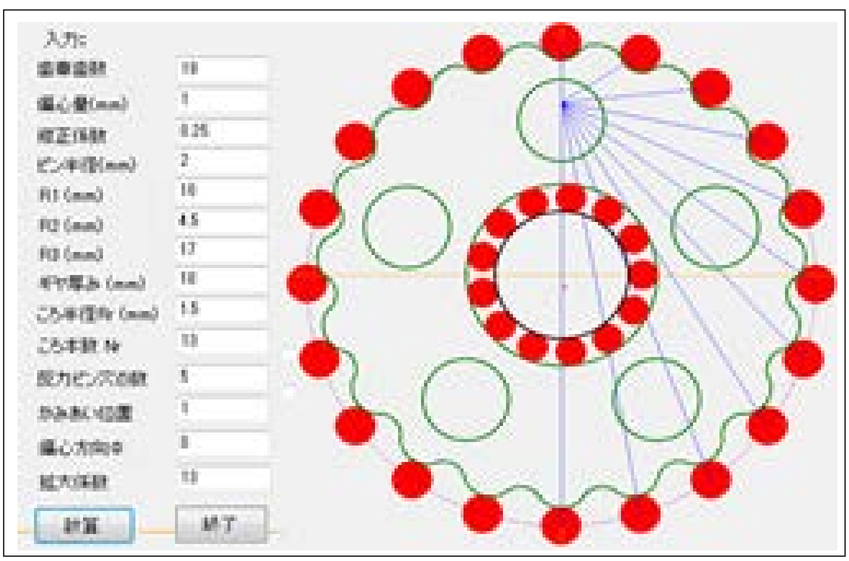

Figure 9 Automatic design program for TGR.

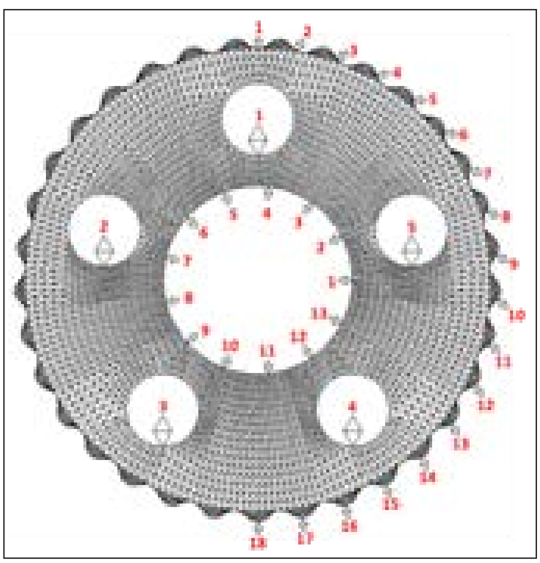

Figure 10 FEM model for contact analysis.

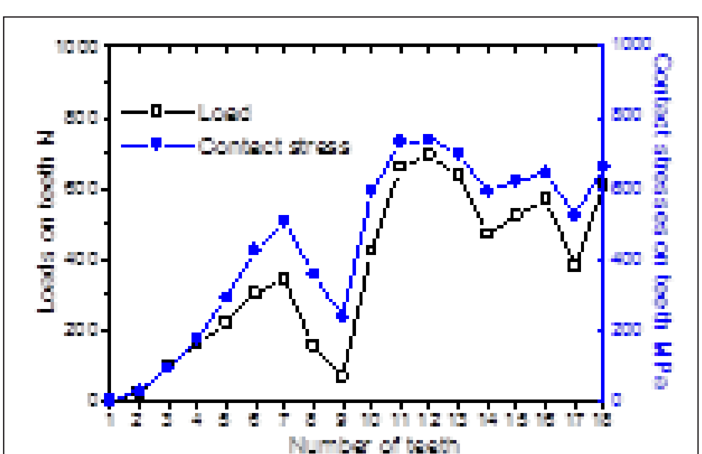

Figure 11 Loads and stresses on teeth.

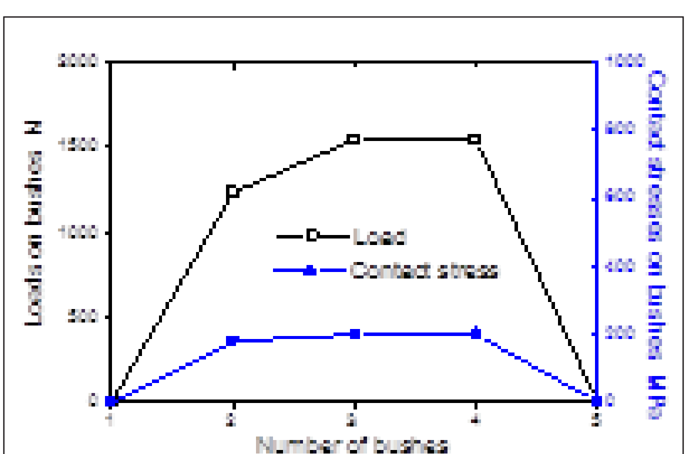

Figure 12 Loads and stresses on bushes.

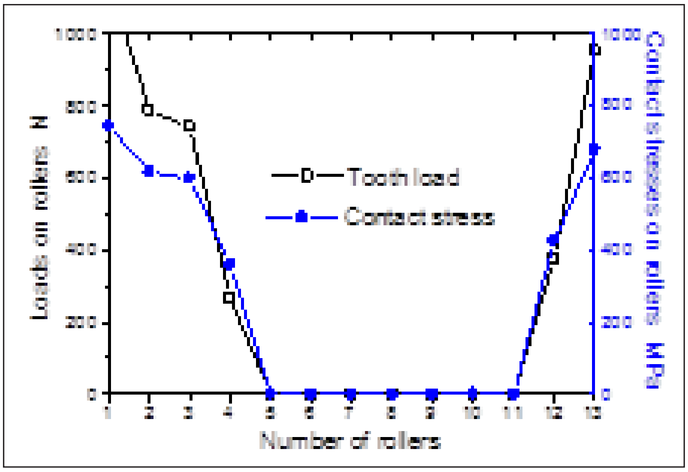

Figure 13 Loads and stresses on rollers.

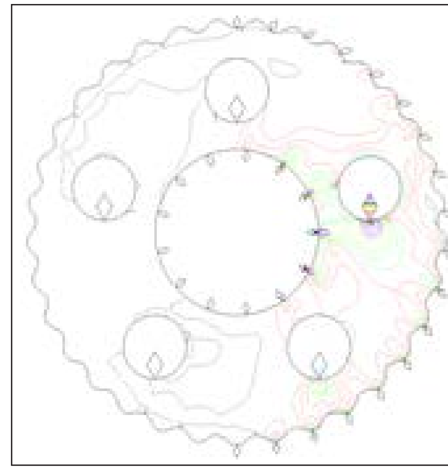

Figure 14 Von Mises stresses.

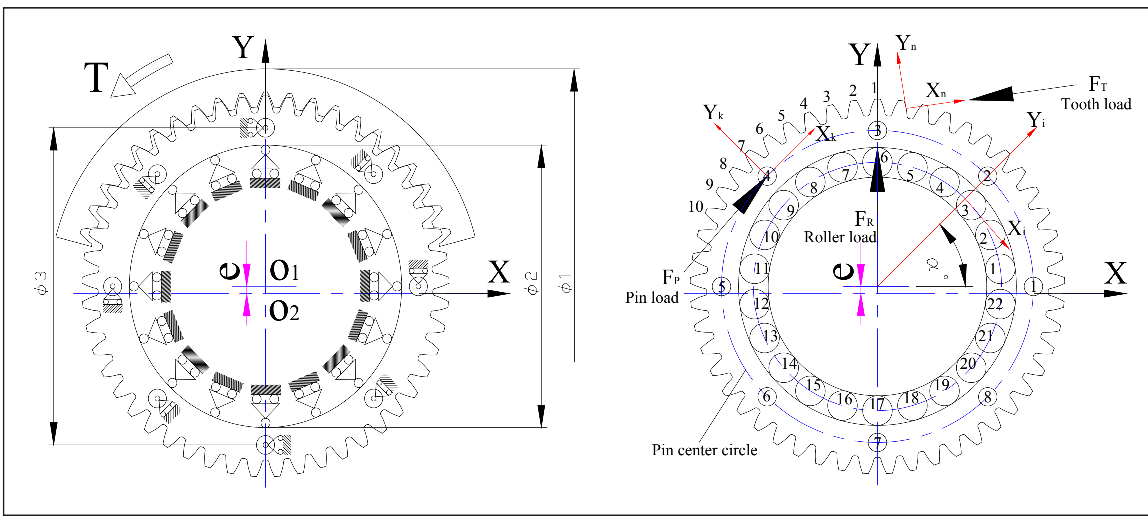

Figure 15 Mechanics model used for contact analysis of the HGR.

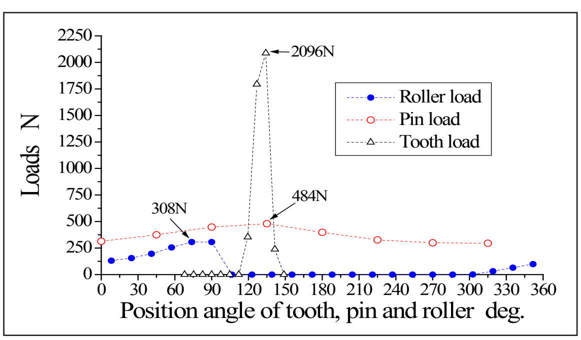

Figure 16 Loads on rollers, pins and teeth.

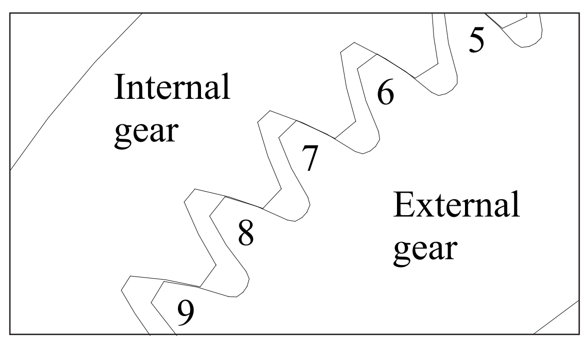

Figure 17 Enlarged view of engaged teeth.

evaluated two-dimensionally. Figure 14 is the calculated Von Mises stresses on the trochoidal gear; it can be used to evaluate the bending strength of the trochoidal gear — but it is rarely used for the TGR.

Hypocyclic gear reducer (HGR). Geometric design and strength calculation of the HGR also differ from usual gear devices. In order to be able to conduct strength calculations of the HGR, a special method used for contact analysis of the HGR has been developed using FEM (Ref. 5). Figure 15 is the mechanics model used for contact analysis of the HGR (Fig. 3). With this model and the developed FEM, loads on contact teeth, pins and center rollers were analyzed (Ref. 5) and are given in Figure 16, where it is found that only four pairs of teeth (tooth pairs 5, 6, 7 and 8 as shown in Figs. 17 and 15) are in contact. This means that the contact tooth number of the HGR is much less than that of the TGR. As shown in Figure 11 there are 17 teeth in contact — from No. 2 to 18 — for the TGR. It also means that tooth profile has a significant effect on contact tooth number. Though the HGR can be made easily because of using the involute profile, since contact teeth are much less than that of the TGR, the load-carrying capacity of the HGR becomes much lower than that of the TGR. This may be why the HGR cannot find wider applications in industry. For now, it is a research sub-

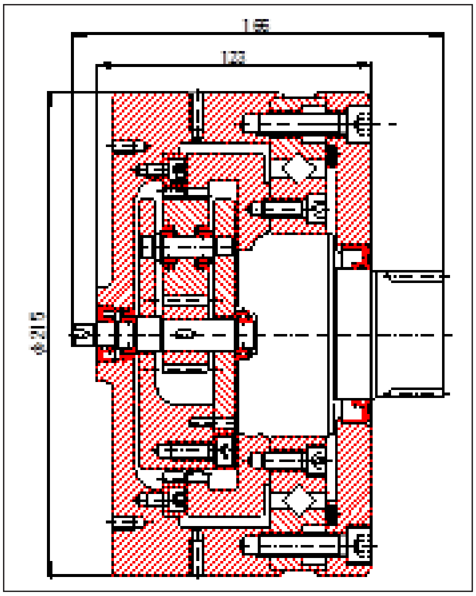

Figure 18 **An example of JFD.**

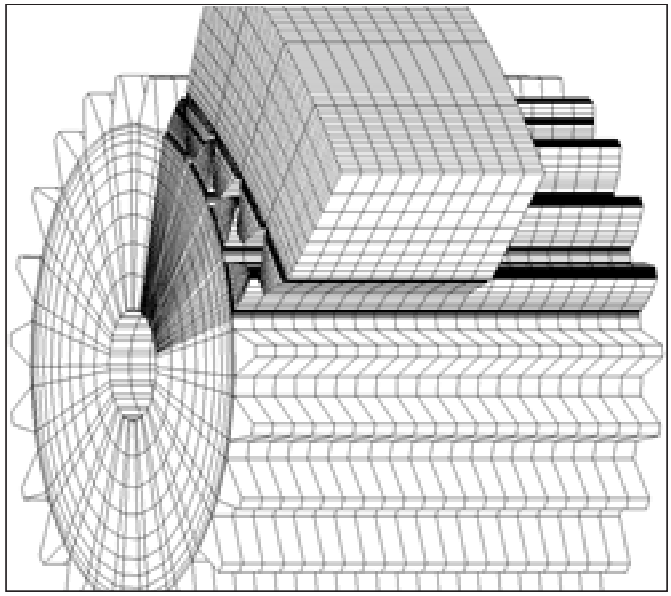

Figure 19 **FEM model (different face widths).**

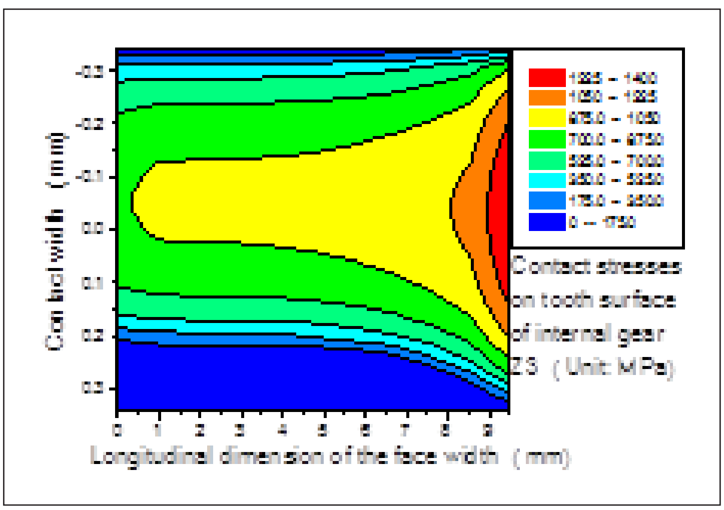

Figure 20 **Tooth contact stresses on Z_3.**

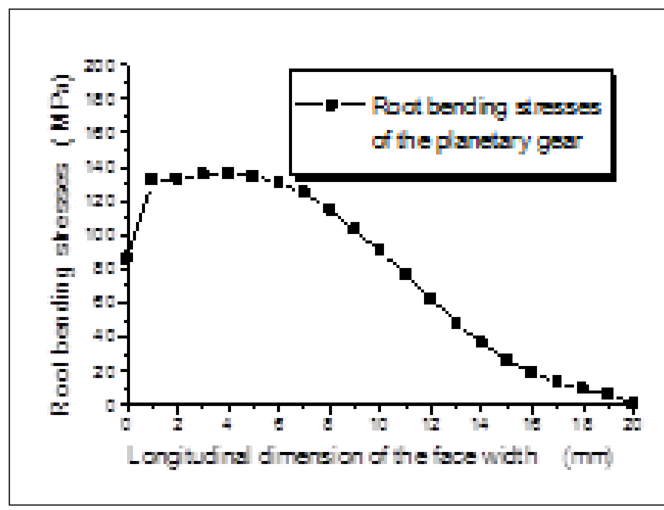

Figure 21 **Tooth root stress of Z_2.**

ject for the author on how to increase the contact tooth number of the HGR; a new tooth profile is under study.

James Ferguson-type planetary drive (JFD). One of structural differences between the JFD and a usual planetary drive is the face width difference of the contact teeth between the planetary gears and the internal gears. In the case of JFD, face width of the planetary gears is about twice that of the internal gears (Fig. 4). This means that only half-a-length of the face width of the planetary gears is used to contact with the entire length of the face width of the internal gears (Z_3 or Z_4). This tooth contact on different face widths makes it difficult to do bending and contact strength calculations of the contact teeth using the methods available for usual gears. So, a 3D FEM is developed to do loaded tooth contact analysis of the contact teeth with different face widths.

Figure18 is a structural example of the JFD designed by the author based on the basic structure shown in Figure 4. In Figure 18, Z_1, Z_2, Z_3 and Z_4 are tooth numbers of the sun gear, planetary gear, and internal gears (left-side and right-side), as indicated in Figure 4. Figure 19 is an FEM model used for loaded tooth contact analysis of the planetary gear Z_2 with the internal gear Z_3. As we see in Figure 19, the internal gear teeth are in contact with the right half-length of the planetary gear teeth.

Figure 20 shows the calculated contact stresses distributed on the tooth surface of the internal gear Z_3; Figure 21 is the calculated root bending stresses of the planetary gear Z_2 distributed along the longitude. From Figure 20 it is determined that contact stresses are not uniformly distributed along the longitude of the internal gear teeth; i.e.—the side-heavier contact occurred on the right side of the contact teeth. In Figure 21 it is also seen that tooth root bending stresses are not uniformly distributed along the longitude of the planetary gear tooth; the left half-length of the planetary gear tooth has larger root stress distribution than the right half-length of the teeth. Although the right half-length of the face width is not used to contact with the internal gear teeth, the right half-length of the tooth also has smaller root stresses. Figures 20 and 21 indicate that the contact and bending stresses of a pair of gears with different face widths are different from that of a pair of normal use gears with equal face widths. It is now another research project for the author on how to reduce the side-heavier contact of a pair of gears with different face widths through lead modification on the internal gear Z_3.

Future Tendencies

A common gearing theory. In the future it will become necessary to build a common gearing theory for the SWGs, TGRs and HGRs, since there is a common point existing in all three devices that use eccentric movement to realize the great transmission ratios, as well as having very similar tooth engagement movements.

To date, gearing principles for a pair of conventional gears without eccentric movements (for example: a pair of spur gears or helical gears) have been well studied, but the same cannot be said for the gearing principles of the GRG with eccentric movements. The gearing theories of the SWG and TGR were built separately, while the gearing theory of the HGR has yet to be built. Plainly, it remains a difficult task to design a new type of tooth profile for the HGR. A most worthwhile future goal is to build a common gearing theory for these three GRGs. It may yet be proven that basic performances and load-carrying capacities of the three GRGs can be greatly improved when a new type of tooth profile is developed with the help of common gearing theory.

Lightweight, small size and high-torque transmission. In the last century, much effort was exerted in making the GRG as lightweight, small-sized and high in load-carrying capacity as possible, primarily because these items were required by users as product specifications and these items reflect the competitiveness of the makers' products. In order to create lightweight, small-size, high-torque transmission, many studies were conducted on gear design, materials, machining accuracy and heat treatment methods. No doubt, these studies markedly improved performance of the GRG and also increased strength and competitiveness of the makers' products. Some makers were very successful in their business and able to compete in very large markets through these efforts. Per the industry experiences of the author, lightweight, small-size and high-torque transmission design are also important for the GRG in the future, as market demand will ensure their importance for quite a long time.

High transmission ratios. Prior to 2000, it was well-accepted to use high-speed, small-size motors in combination with the GRG to transmit great torque in industry robots and other machines. This idea is also successful in humanoid robots (for an example, Honda's ASIMO), NASA's Mars Rover and many other new machines. Now this idea is accepted by many engineers as a general design thought and used widely in new machine development. A familiar nickname — "Gearhead" — is also given to this idea by engineers, and is used very often in factories. Today's "Gearheads" often have transmission ratios among 30 – 200. In the future, much higher transmission ratios — among 200 – 400, for example — may well be needed in the marketplace. This would create new business opportunities in the future to design and make the much higher transmission ratio "Gearheads."

High performances. Also prior to 2000 much effort was expended in improving the performance of the GRG in order to satisfy market requirements. Today it is possible to make very high-performance GRGs with low transmission errors, low vibration, low noise, high transmission efficiency, and high strength with the help of gear technology advances in design, FEM analysis, materials, heat treatment and machining accuracy. Per the author's industry experience, it can be predicted that even much higher GRG performance will be pursued by users in the future.

Conclusions

- Structures, advantages and basic performance of four types of great transmission ratio gear devices — the strain wave gearing device; trochoidal gear reducer; hypocyclic gear reducer; and James Ferguson-type of planetary drive are introduced and compared.
- The latest design and strength analysis methods of the four great transmission ratio gear devices are introduced.
- Future tendencies of the great transmission ratio gear devices are predicted, based upon the extensive industry experience of the author with great ratio gear devices. **PTE**

References

1. Musser, C. W. "Breakthrough in Mechanical Drive Design — The Harmonic Drive," *Machine Design*, April 14, pp. 160-173, 1960.
2. Ferguson, James. The Description and Use of a New Machine, Called the Mechanical Paradox; Invented by James Ferguson, London, 1764.
3. Takashi, N. *Involute Profile Shifted Gears* (in Japanese), JSME press, 1994.
4. (4) Li, S. "A Challenge to Design of a New Harmonic Drive Device," *Applied Mechanics and Materials*, Vol. 86, 43-46, 2011.
5. Li, S. "Contact Problem and Numeric Method of a Planetary Drive with Small Teeth Number Difference," *Mechanism and Machine Theory*, Vol.43, Issue 9, 1065-1086, 2008.

Shuting Li is an associate professor at Shimane University in Japan, and an active member of the Administration Committee of the JSME Machine Design & Tribology Division. He graduated in 1986 from the Department of Aircraft Manufacturing at the Northwestern Polytechnical University in China, receiving his Ph.D. from Yamaguchi University (Japan) in 1998. Li has 28 years' experience in gear research — including 13 years of industry experience. He has authored more than 50 papers on gear research and since 2000 has also served as reviewer for many international journals, including ASME Trans. J. Mech. Des., and Mech. Mach. Theory.