GYANTECH GURU JI

Mathematics

Class 10 (Secondary) — Handwritten Notes

Full Syllabus | Chapter-wise | Formulas & Examples | Important Topics

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SAMPLE – 1 chapter

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10 **Quadratic Equations** ★★★★★

How to use Read the notes chapter by chapter, copy the red boxes in your own notebook and practise every example yourself. Stars show how important a chapter is for the exam.

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Chapter 10

Quadratic Equations

Importance: ★★★★★ Factorisation, formula, discriminant, word problems

10.1 What is a Quadratic Equation?

A polynomial of **degree 2** is a quadratic polynomial. When we put it equal to zero, we get a quadratic equation. Only one variable (x) is used in this chapter.

Standard form

$ax^2 + bx + c = 0$, $a \neq 0$

a = coefficient of x^2 , b = coefficient of x , c = constant term

- Highest power of x must be **exactly 2**. If $a = 0$ it becomes linear.
- Always write terms in **descending order** (x^2 first, then x, then constant).
- A quadratic equation has **at most two roots**.
- Brahmagupta gave an explicit formula; **Sridharacharya** (c. 1025 CE) gave the quadratic formula, quoted by Bhaskara-II.

Ex. **Is $(x + 5)(x - 4) = 0$ a quadratic equation? Write standard form.**

Multiply: $x^2 - 4x + 5x - 20 = 0$

$\Rightarrow x^2 + x - 20 = 0$ (degree 2, so **Yes**)

Ex. **Is $x + 1/x = 6$ a quadratic equation?**

Multiply by x: $x^2 + 1 = 6x$

$\Rightarrow x^2 - 6x + 1 = 0$ (degree 2, so **Yes**)

Note $x^3 + 2x = 0$ is NOT quadratic (degree 3). $7x - 9 = 0$ is NOT quadratic (degree 1). Always simplify first, then check the degree.

10.2 Root of an Equation

A value of x which makes **LHS = RHS** is called a **root** (or zero). If $x = \alpha$ satisfies the equation, then α is a root.

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Ex. **Is 1 a root of $x^2 - 5x + 6 = 0$?**

Put $x = 1$: $1 - 5 + 6 = 2 \neq 0 \Rightarrow 1$ is **not** a root.

Put $x = 2$: $4 - 10 + 6 = 0 \Rightarrow 2$ **is** a root.

10.3 Method 1 – Factorisation

Write the equation as a product of two linear factors, then use: **if A × B = 0, then A = 0 or B = 0.**

Splitting the middle term

1. Multiply $a \times c$.
2. Find two numbers whose product = $a \times c$ and sum = b .
3. Split bx into those two terms, group and take common factors.
4. Put each factor = 0 and solve.

Ex. **Solve $x^2 - 2x - 24 = 0$**

$a \times c = -24$; numbers -6 and 4 (sum -2)

$x^2 - 6x + 4x - 24 = 0$

$x(x - 6) + 4(x - 6) = 0$

$(x - 6)(x + 4) = 0 \Rightarrow x = 6$ or $x = -4$

Ex. **Solve $4x^2 - 12x + 9 = 0$**

This is a perfect square: $(2x)^2 - 2(2x)(3) + 3^2 = (2x - 3)^2$

$(2x - 3)^2 = 0 \Rightarrow x = \frac{3}{2}$ (twice)

Vedic Maths – Urdhva-Tiryagbhyam (cross-multiplication)

Factorise the first term and the last term.

Check: the sum of the cross-products must equal the middle term.

Example: $6x^2 + 52x + 46 = 0 \rightarrow (2x)(3x)$ and $(2)(23)$

cross-product: $2x \times 23 + 3x \times 2 = 52x$ ✓

so $(2x + 2)(3x + 23) = 0 \Rightarrow x = -1$ or $x = -\frac{23}{3}$

10.4 Method 2 – Quadratic Formula

Multiply $ax^2 + bx + c = 0$ by $4a$ and add b^2 on both sides. We get $(2ax + b)^2 = b^2 - 4ac$. Taking the square root gives the formula.

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Quadratic Formula (Sridharacharya)

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

Roots: $\alpha = \frac{-b + \sqrt{D}}{2a}$, $\beta = \frac{-b - \sqrt{D}}{2a}$

The expression $b^2 - 4ac$ is called the Discriminant and is written as D.

Ex. **Solve $4x^2 + 12x + 5 = 0$ by formula**

$a = 4$, $b = 12$, $c = 5$

$D = b^2 - 4ac = 144 - 80 = 64$

$x = \frac{-12 \pm \sqrt{64}}{2 \times 4} = \frac{-12 \pm 8}{8}$

$x = \frac{-4}{8} = -\frac{1}{2}$ or $x = \frac{-20}{8} = -\frac{5}{2}$

Note Take care of signs: if b is negative, −b becomes positive. Always write a, b, c first with their signs.

10.5 Nature of Roots

Discriminant $D = b^2 - 4ac$

$D > 0 \Rightarrow$ **two real and distinct** roots

$D = 0 \Rightarrow$ **two real and equal** roots, $x = -\frac{b}{2a}$

$D < 0 \Rightarrow$ **no real roots** (square root of a negative number is not real)

Ex. **Nature of roots of $2x^2 + 16x + 25 = 0$**

$a = 2$, $b = 16$, $c = 25$

$D = 256 - 4(2)(25) = 256 - 200 = 56$

$D = 56 > 0 \Rightarrow$ **real and distinct roots**

Ex. **For what value of k does $x^2 + 4x + k = 0$ have real distinct roots?**

$D = 16 - 4k > 0$

$\Rightarrow 16 > 4k \Rightarrow k < 4$

10.6 Word Problems

1. Read the problem twice and choose the unknown as **x**.
2. Form the equation from the given statement.
3. Solve it by factorisation or by formula.

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4. **Reject** answers that do not fit (length, age or number of things cannot be negative).
5. Verify the answer with the problem.

Ex. **The product of two consecutive positive integers is 306. Find them.**

Let the numbers be x and $x + 1 \Rightarrow x(x + 1) = 306$

$x^2 + x - 306 = 0 \Rightarrow x^2 + 18x - 17x - 306 = 0$

$(x + 18)(x - 17) = 0 \Rightarrow x = -18$ **(rejected)** or $x = 17$

The integers are 17 and 18.

Ex. **A rectangle is 8 cm longer than it is wide and its area is 240 cm². Find the sides.**

Width = x, length = $x + 8 \Rightarrow x(x + 8) = 240$

$x^2 + 8x - 240 = 0 \Rightarrow (x + 20)(x - 12) = 0$

$x = 12$ (width, since -20 is rejected)

Length = 20 cm, Width = 12 cm

Quick Revision

Remember for the exam

Standard form: $ax^2 + bx + c = 0$, $a \neq 0$

Sum of roots $= -\frac{b}{a}$ and product of roots $= \frac{c}{a}$

$D = b^2 - 4ac$ decides the nature of roots

Formula: $x = \frac{-b \pm \sqrt{D}}{2a}$

Always reject a negative length / age / number in word problems

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Topic-wise Important Topics

Expected / important topics from our analysis. Not a guarantee of the actual paper.

Ch 10 – Quadratic Equations

- ★ ★★★★★ Standard form and checking whether an equation is quadratic
- ★ ★★★★★ Solving by factorisation (splitting the middle term)
- ★ ★★★★★ Quadratic formula and the discriminant
- ★ ★★★★★ Nature of roots and finding the value of k
- ★ ★★★★☆ Word problems: consecutive numbers, area, age, numerator-denominator
- ★ ★★★☆☆ Vedic Mathematics method (Urdhva-Tiryagbhyam)

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